Essay

Secret Link Between Pure Math & Physics Uncovered

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Abstract

Number theorist Minhyong Kim has speculated about very interesting general connection between number theory and physics. The reading of a popular article about Kim's work revealed that number theoretic vision about physics provided by TGD has led to a very similar ideas and suggests a concrete realization of Kim's ideas. In the following I briefly summarize what I call identification problem. The identification of points of algebraic surface with coordinates, which are rational or in extension of rationals, is in question. In TGD framework the imbedding space coordinates for points of space-time surface belonging to the extension of rationals defining the adelic physics in question are common to reals and all extensions of p-adics induced by the extension. These points define what I call cognitive representation, whose construction means solving of the identification problem. Cognitive representation defines discretized coordinates for a point of "world of classical worlds" (WCW) taking the role of the space of spaces in Kim's approach. The symmetries of this space are proposed by Kim to help to solve the identification problem. The maximal isometries of WCW necessary for the existence of its Kähler geometry provide symmetries identifiable as symplectic symmetries. The discrete subgroup respecting extension of rationals acts as symmetries of cognitive representations of space-time surfaces in WCW, and one can identify symplectic invariants characterizing the space-time surfaces at the orbits of the symplectic group.

1 Introduction

I learned about a possible existence of a very interesting link between pure mathematics and physics (see `http://tinyurl.com/y86bckmo`). The article told about ideas of number theorist Minhyong Kim working at the University of Oxford. As I read the popular article, I realized it is something very familiar to me but from totally different view point.

Number theoretician encounters the problem of finding rational points of an algebraic curve defined as real or complex variant in which case the curve is 2-D surface and 1-D in complex sense. The curve is defined as root of polynomials polynomials or several of them. The polynomial have typically rational coefficients but also coefficients in extension of rationals are possible.

For instance, Fermat's theorem is about whether $x^n + y^n = 1$, $n = 1, 2, 3, ...$ has rational solutions for $n \geq 1$. For $n = 1$, and $n = 2$ it has, and these solutions can be found. It is now known that for $n > 2$ no solutions do exist. Quite generally, it is known that the number is finite rather than infinite in the generic case.

A more general problem is that of finding points in some algebraic extension of rationals. Also the coefficients of polynomials can be numbers in the extension of rationals. A less demanding problem is mere counting of rational points or points in the extension of rationals and a lot of progress has been achieved in this problem. One can also dream of classifying the surfaces by the character of the set of the points in extension.

1.1 Rational points for algebraic curves

I have considered the identification problem earlier in [10] and I glue here a piece of text summarizing some basic results. The generic properties of sets of rational points for algebraic curves are rather well

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understood. Mordelli conjecture proved by Falting as a theorem (see http://tinyurl.com/y9oq37ce) states that a curve over Q with genus $g = (d-1)(d-2)/2 > 1$ (degree $d > 3$) has only finitely many rational points.

1. Sphere CP_1 in CP_2 has rational points as a dense set. Quite generally rational surfaces, which by definition allow parametric representation using polynomials with rational coefficients (encountered in context of Du Val singularities characterized by the extended Dynkin diagrams for finite subgroups of $SU(2)$) allow dense set of rational points [3, 6]).

   $g = 0$ does not yet guarantee that there is dense set of rational points. It is possible to have complex conics (quadratic surface) in CP_2 with no rational points. Note however that this depends on the choice of the coordinates: if origin belongs to the surface, there is at least one rational point

2. Elliptic curve $y^2 - x^3 - ax - b$ in CP_2 (see http://tinyurl.com/lovksny) has genus $g = 1$ and has a union of lattices of rational points and of finite cyclic groups of them since it has origin as a rational point. This lattice of points are generated by translations. Note that elliptic curve has no singularities that is self intersections or cusps (for $a = 0, b = 0$ origin is a singularity).

   $g = 1$ does not guarantee that there is infinite number of rational points. Fermat's last theorem and CP_2 as example. $x^d + y^d = z^d$ is projectively invariant statement and therefore defines a curve with genus $g = (d-1)(d-2)/2$ in CP_2 (one has $g = 0, 0, 2, 3, 6, 10, ...$). For $d > 2$, in particular $d = 3$, there are no rational points.

3. $g \geq 2$ curves do not allow a dense set of rational points nor even potentially dense set of rational points.

In my article [10] providing TGD perspective about the role of algebraic geometry in physics, one can find basic results related to the identification problem including web links and references to literature.

1.2 TGD variant of the key idea of Kim

How to identify the sets of rational or more general points in the extension of rationals defining in TGD framework a cognitive representation as points common to reals representing the physics of sensory experience and various p-adic numbers correlating with the physics of cognition and imagination [8, 12, 10]?

1. In modern physics symmetries are indispensable: could symmetries come in rescue also now and could there be a connection with physics? Here Kim introduces the notion of space of spaces, whose symmetries might allow to organize the understanding of these sets into a bigger picture. In TGD framework the extension of physics to describe also the correlates of cognition has led to the identification problem but also to the identification of symmetries possible relevant for understanding this problem.

2. In string models the space of gauge field configurations consider by Kim would be replaced by string world sheets (loop space does not allow to realize 2-D general coodrinate invariance). In TGD framework one would have space of space-time surfaces instead of gauge configurations. In TGD framework the points of space-time surface in preferred imbedding coordinates (this requires maximal isometries for the imbedding space) belonging to a extension of rationals defining the adelic physics in question are common to reals and all extensions of p-adics induced by extension of rationals [11, 12, 10]. These space-time points define what I call cognitive representation and their finding is just the identification problem. Cognitive representation defines discretized coordinates for a point of "world of classical worlds" (WCW) replacing the space of spaces in Kim's approach.

3. The symmetries of the space of spaces are proposed by Kim to help to solve the identification problem. Indeed, the maximal isometries of WCW necessary for the existence of its Kähler geometry would provide these symmetries and they would act as symplectic symmetries. The discrete subgroup of the symplectic group respecting the extension of rationals defining the adelic physics acts as symmetries in the space of the cognitive representations of surfaces in WCW - discretized variant of WCW -, and one can identify symplectic invariants characterizing the space-time surfaces at the orbits of the symplectic group.

2 Connection of the identification problem with TGD and physics of cognition

The key TGD inspired idea is already described: number theory provides the mathematics needed for the understanding of correlates of cognition and cognitive representations in turn provide ideal models of physical systems as the astonishing success of p-adic mass calculations suggests [7]. Physics can however also help number theory via the power of huge symmetries of unified theories.

2.1 Cognitive representations number theoretically

The identification problem is extremely difficult even for mathematicians - to say nothing about humble physicist like me with hopelessly limited mathematical skills. It is however just this problem which I encounter in TGD inspired vision about adelic physics [11, 12, 10]. Recall that in TGD space-times are 4-surfaces in $H = M^4 \times CP_2$, preferred extremals of the variational principle defining the theory [9, 13].

1. In this approach p-adic physics for various primes p provide the correlates for cognition: there are several motivations for this vision. Ordinary physics describing sensory experience and the new p-adic physics describing cognition for various primes p are fused to what I called adelic physics. The adelic physics is characterized by extension of rationals inducing extensions of various p-adic number fields. The dimension n of extension characterizes kind of intelligence quotient and evolutionary level since algebraic complexity is the larger, the larger the value of n is. The connection with quantum physics comes from the conjecture that n is essentially effective Planck constant $h_{eff}/h_0 = n$ characterizing a hierarchy of dark matters. The larger the value of n the longer the scale of quantum coherence and the higher the evolutionary level, the more refined the cognition.

2. An essential notion is that of cognitive representation [8, 12, 10]. It has several realizations. One of them is the representation as a set of points common to reals and extensions of various p-adic number fields induced by the extension of rationals. These space-time points have points in the extension of rationals considered defining the adele. The coordinates are the imbedding space coordinates of a point of the space-time surface. The symmetries of imbedding space provide highly unique imbedding space coordinates.

3. The gigantic challenge is to find these points common to real number field and extensions of various p-adic number fields appearing in the adele.

4. If this were not enough, one must solve an even tougher problem. WCW [9] consists of space-time surfaces in imbedding space $H = M^4 \times CP_2$, which are so called preferred extremals of the action principle of theory. Quantum physics would reduce to geometrization of WCW and construction of classical spinor fields in WCW and representing basically many-fermion states: only the quantum jump would be genuinely quantal in quantum theory.

   There are good reasons to expect that space-time surfaces are minimal surfaces with 2-D singularities, which are string world sheets - also minimal surfaces [13, 14]. This gives nice geometrization of gauge theories since minimal surfaces equations are geometric counterparts for massless field equations.

One must find the algebraic points, the cognitive representation, for all these preferred extremals representing points of WCW (one must have preferred coordinates for H - the symmetries of imbedding space crucial for TGD and making it unique, provide the preferred coordinates)!

5. What is so beautiful is that in given cognitive resolution defined by the extension of rationals inducing the discretization of space-time surface, the cognitive representation defines the coordinates of the space-time surfaces as a point of WCW. In finite cognitive and measurement resolution this huge infinite-dimensional space WCW discretizes and the situation can be handled using finite mathematics.

2.2 Connection of identification problem with Kim's work

So: what is then the connection with the work and ideas of Kim. There has been a lot of progress in understanding the problem: here I an only refer to the popular article.

1. One step of progress has been the realization that if one uses the fact that the solutions are common to both reals and various p-adic number fields helps a lot. The reason is that for rational points the rationality implies that the solution of equation representable as infinite power series of p contains only finite number powers of p. If one manages to prove the this happens for even single prime, a rational solution has been found.

   The use of reals and all p-adic numbers fields is nothing but adelic physics. Real surfaces and all its p-adic variants form pages of a book like structure with infinite number of pages. The rational points or points in extension of rationals are the cognitive representation and are points common to all pages in the back of the book.

   This generalizes also to algebraic extensions of rationals. Solving the number theoretic problem is in TGD framework nothing but finding the points of the cognitive representation. The surprise for me was that this viewpoint helps in the problem rather than making it more complex.

   There are however problematic situations in some cases the hypothesis about finite set of algebraic points need not make sense. A good example is Fermat for $x + y = 1$. All rational points and also algebraic points are solutions. For $x^2 + y^2 = 1$ the set of Pythagorean triangles characterizing the solutions is infinite. How to cope with these situations in which one has accidental symmetries as one might say?

2. Kim argues that one can make even further progress by considering the situation from even wider perspective by making the problem even bigger. Introduce what popular article calls the space of spaces. The space of string world sheets is what string models suggests. WCW is what TGD suggests. One can get a wider perspective of the problem of finding algebraic points of a surface by considering the problem in the space of surfaces and at this level it might be possible to gain much more understanding. The notion of WCW would not mean horrible complication of a horribly complex problem but possible manner to understand the problem!

   The popular article mentioned in the beginning mentions so called Selmer varieties as a possible candidate for the space of spaces. From the Wikipedia article (see `http://tinyurl.com/y27so3f2`) telling about Kim one can find a link to an article [4] related to Selmer varieties. This article goes over my physicist's head but might give for a more mathematically oriented reader some grasp about what is involved. One can find also a list of publications of Kim (see `http://people.maths.ox.ac.uk/kimm/`.

Kim also suggests that the spaces of gauge field configurations could provide the spaces of spaces. The list contains an article [5] with title *Arithmetic Gauge Theory: A Brief Introduction* (see `http://tinyurl.com/y66mphkh`) , which might help physicist to understand the ideas. An arithmetic variant of gauge theory could provide the needed space of spaces.

2.3 Can one make Kim's idea about the role of symmetries more concrete in TGD framework?

The crux of the Kim's idea is that somehow symmetries of space of spaces could come in rescue in the attempts to understand the rational points of surface. The notion of WCW suggest in TGD framework rather concrete realization of this idea that I have discussed from the point of view of construction of quantum states.

1. A little bit more of zero energy ontology (ZEO) is needed to follow the argument. In ZEO causal diamonds (CDs) are central. CDs are defined as intersections of future and past directed light-cones with points replaced with CP_2 and forming a scale hierarchy are central. Space-time surfaces are preferred extremals with ends at the opposite boundaries of CD indeed looking like diamond. Symplectic group for the boundaries of causal diamond (CD) is the group of isometries of WCW [9, 13]. Maximal isometry group is required to guarantee that the WCW Kähler geometry has Riemann connection - this was discovered for loop spaces by Dan Freed [1]. Its Lie algebra has structure of Kac- Moody algebra with respect to the light-like radial coordinate of the light-like boundary of CD, which is piece of light-cone boundary. This infinite-D group plays central role in quantum TGD: it acts as maximal group of WCW isometries and zero energy states are invariant under its action at opposite boundaries.

2. As one replaces space-time surface with a cognitive representation associated with an extension of rationals, WCW isometries are replaced with their infinite discrete subgroup acting in the number field define by the extension of rationals defining the adele. These discrete isometries do not leave the cognitive representation invariant but replace with it new one having the same number of points and one obtains entire orbit of cognitive representations. This is what the emergence of symmetries in wider conceptual framework would mean.

3. One can in fact construct invariants of the symplectic group. Symplectic transformations leave invariance the Kähler magnetic fluxes associated with geodesic polygons with edges identified as geodesic lines of H. There are also higher-D symplectic invariants. The simplest polygons are geodesic triangles. The symplectic fluxes associated with the geodesic triangles define symplectic invariants characterizing the cognitive representation. For the twistor lift one must allow also M^4 to have analog of Kähler form - it would be responsible for CP violation and matter antimatter asymmetry [15]. Also this defines symplectic invariants so that one obtains them for both M^4 and CP_2 projections and can characterize the cognitive representations in terms of these invariants. Note that the existence of twistor lift fixes the choice of H uniquely since M^4 and CP_2 are the only 4-D spaces allowing twistor space with Kähler structure [2] necessary for defining the twistor lift of Kähler action.

   More complex cognitive representations in an extension containing the given extension are obtained by adding points with coordinates in the larger extension and this gives rise to new geodesic triangles and new invariants. A natural restriction could be that the polynomial defining the extension characterizing the preferred extremal via $M^8 - H$ duality defines the maximal extension involved.

4. Also in this framework one can have accidental symmetries. For instance, M^4 with CP_2 coordinates taken to be constant is a minimal surface, and all rational and algebraic points for given extension belong to the cognitive representation so that they are infinite. Could this has something to do with the fact that we understand M^4 so well and have even identified space-time with Minkowski

space! Linear structure would be cognitively easy for the same reason and this could explain why we must linearize.

CP_2 type extremals with light-like M^4 geodesic as M^4 projection is second example of accidental symmetries. The number of rational or algebraic points with rational M^4 coordinates at light-like curve is infinite - the situation is very similar to $x + y = 1$ for Fermat. Simplest cosmic strings are geodesic sub-manifolds, that is products of plane $M^2 \subset M^4$ and CP_2 geodesic sphere. Also they have exceptional symmetries.

What is interesting from the point of view of proposed model of cognition is that these cognitively easy objects play a central role in TGD: their deformations represent more complex dynamical situations. For instance, replacing planar string with string world sheet replaces cognitive representation with a discrete or perhaps even finite one in M^4 degrees of freedom.

5. A further TGD based simplification would be $M^8 - H$ $(H = M^4 \times CP_2)$ duality in which space-time surfaces at the level of M^8 are algebraic surfaces, which are mapped to surfaces in H identified as preferred extremals of action principle by the $M^8 - H$ duality [10]. Algebraic surfaces satisfying algebraic equations are very simple as compared to preferred extremals satisfying partial differential equations but "preferred" is what makes possible the duality. This huge simplification of the solution space of field equations guarantees holography necessitated by general coordinate invariance implying that space-time surfaces are analogous to Bohr orbits. It would also guarantee the huge symmetries of WCW making it possible to have Kähler geometry.

   This suggests in TGD framework that one finds the cognitive representation at the level of M^8 using methods of algebraic geometry and maps the points to H by using the $M^8 - H$ duality. TGD and octonionic variant of algebraic geometry would meet each other.

   It must be made clear that now solutions are not points but 4-D surfaces and this probably means also that points in extension of rationals are replaced with surfaces with imbedding space coordinates defining function in extensions of rational functions rather than rationals. This would bring in algebraic functions. This might provide also a simplification by providing a more general perspective. Also octonionic analyticity is extremely powerful constraint that might help.

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References

[1] Freed DS. The Geometry of Loop Groups, 1985.

[2] N. Hitchin. Kählerian twistor spaces. *Proc London Math Soc.* Available at: `http://tinyurl.com/pb8zpqo`, 8(43):133–151, 1981.

[3] McKay J. Cartan matrices, finite groups of quaternions, and kleinian singularities. *Proc AMS.* *http://tinyurl.com/ydygjgge*, 1981.

[4] Coates J and Kim M. Selmer varieties for curves with CM Jacobians. *Kioto J Math.* Available at: `http://people.maths.ox.ac.uk/kimm/papers/cm3.pdf`, 50(4):827–852, 2010.

[5] Kim M. Arithmetic Gauge Theory: A Brief Introduction. *Mod Phys Lett A.*Available at: `https://www.worldscientific.com/doi/10.1142/S0217732318300124`, 33(29), 2018.

[6] Reid M. The du val singularities an, dn, e6, e7, e8. Available at:`http://homepages.warwick.ac.uk/~masda/surf/more/DuVal.pdf`.

[7] Pitkänen M. Massless states and particle massivation. In *p-Adic Physics*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/padphys.html#mless, 2006.

[8] Pitkänen M. p-Adic Physics as Physics of Cognition and Intention. In *TGD Inspired Theory of Consciousness*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdconsc.html#cognic, 2006.

[9] Pitkänen M. Recent View about Kähler Geometry and Spin Structure of WCW . In *Quantum Physics as Infinite-Dimensional Geometry*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdgeom.html#wcwnew, 2014.

[10] Pitkänen M. Does M^8-H duality reduce classical TGD to octonionic algebraic geometry? Available at: http://tgdtheory.fi/public_html/articles/ratpoints.pdf, 2017.

[11] Pitkänen M. Philosophy of Adelic Physics. In *Trends and Mathematical Methods in Interdisciplinary Mathematical Sciences*, pages 241–319. Springer.Available at: https://link.springer.com/chapter/10.1007/978-3-319-55612-3_11, 2017.

[12] Pitkänen M. Philosophy of Adelic Physics. Available at: http://tgdtheory.fi/public_html/articles/adelephysics.pdf, 2017.

[13] Pitkänen M. The Recent View about Twistorialization in TGD Framework. Available at: http://tgdtheory.fi/public_html/articles/smatrix.pdf, 2018.

[14] Pitkänen M. Minimal surfaces: comparison of the perspectives of mathematician and physicist. Available at: http://tgdtheory.fi/public_html/articles/minimalsurfaces.pdf, 2019.

[15] Pitkänen M. Breaking of CP, P, and T in cosmological scales in TGD Universe. Available at: http://tgdtheory.fi/public_html/articles/rotuniverse.pdf, 2017.